1) Solve the differential equation, \( xy' = y + 1 \), subject to the condition \( y(x=1)=0 \).

2) a) Find, by whatever means you wish, (short of cheating) two linearly independent solutions to the following differential equation.

\[
\frac{d^2y}{dt^2} + y = 0
\]

b) Prove that the two solutions are indeed independent solutions.

3) Use Green's function techniques to solve the following differential equation

\[
\frac{d^2y}{dx^2} = x - y
\]

subject to the following boundary conditions. \( y(x=0)=0 \) and \( y(x=\pi/2)=0 \).

4) Consider the following differential equation.

\[
y'' - y' - \frac{1}{z}y = 0
\]

a) Assume a Frobenius series solution. Determine the roots of the indicial equation.

b) Pick the larger of the two roots and find the series solution associated with it. You are not required to find the second solution.