

Symbol Table  
for Manifolds, Tensors, and Forms

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# 1

## List of Symbols

### 1.1 Rings, Fields, and Spaces

Symbol	Description	Page
$\mathbb{N}$	natural numbers	264
$\mathbb{Z}$	integers	265
$\mathbb{F}$	an arbitrary field	1
$\mathbb{R}$	real field or real line	1
$\mathbb{R}^n$	(real) $n$ space	1
$\mathbb{R}\mathbb{P}^n$	real projective $n$ space	68
$\mathbb{H}^n$	(real) upper half $n$ -space	167
$\mathbb{C}$	complex plane	15
$\mathbb{C}^n$	(complex) $n$ space	15

### 1.2 Unary operations

Symbol	Description	Page
$\bar{a}$	complex conjugate	14
$\bar{X}$	set complement	263
$ x $	absolute value	16
$ X $	cardinality of set	264
$\ x\ $	length of vector	57
$[x]$	equivalence class	264

$f^{-1}(y)$	inverse image of $y$ under $f$	263
$f^{-1}$	inverse map	264
$(-1)^\sigma$	sign of permutation $\sigma$	266
$\star$	hodge dual	45
$\nabla$	(ordinary) gradient operator	73
$\nabla_X$	covariant derivative in direction $X$	182
$d$	exterior derivative	89
$\delta$	coboundary operator (on cohomology)	127
$\delta$	co-differential operator	222
$\Delta$	Hodge-de Rham Laplacian	223
$\nabla^2$	Laplace-Beltrami operator	242
$f^*$	pullback map	95
$f_*$	pushforward map	97
$f_*$	induced map on simplices	161
$i_X$	interior product	93
$\mathcal{L}_X$	Lie derivative	102
$\Sigma X$	suspension	119
$\partial$	partial derivative	59
$\partial$	boundary operator	143
$\partial^*$	coboundary operator (on cochains)	170
$[S]$	simplex generated by set $S$	141
$D$	vector bundle connection	182
$\text{ind}(X, p)$	index of vector field $X$ at $p$	260
$I(f; p)$	index of $f$ at $p$	250

### 1.3 More unary operations

Symbol	Description	Page
Ad	(big) Ad	109
ad	(little) ad	109
alt	alternating map	42
Ann	annihilator	9
Aut	automorphism group	4
cl	closure	56

coker	cokernel	21
deg	degree	251
Deg	degree	251
dim	vector space dimension	2
dim	manifold dimension	63
eu	Euler class	262
im	image	3
ker	kernel	3
mod $n$	modulo $n$	264
pf	Pfaffian	261
rk	rank	3
supp	support	129

#### 1.4 Binary relations

Symbol	Description	Page
$\in$	element of	263
$\subseteq$	subset of	263
$\cong$	isomorphic	3
$\approx$	homeomorphic	58
$\sim$	equivalent	264
$\sim$	homotopic	118

#### 1.5 Binary operations

Symbol	Description	Page
$\cup$	union	263
$\cap$	intersection	263
$\oplus$	direct sum	2
$\circ$	composition	264
$\langle x, y \rangle$	dual pairing	8
$g(u, v)$	inner product	14
$g(u, v)$	(smooth) metric	193
$\otimes$	tensor product (of tensors)	30

$\otimes$	tensor product (of maps)	49
$v \wedge w$	wedge product	38
$\odot$	symmetric tensor product	51
$(X, Y)$	Killing form	110
$X \vee Y$	wedge sum	138
$X \amalg Y$	disjoint union	138
$[X, Y]$	Lie bracket of $X$ and $Y$	77
$\{f, g\}$	Poisson bracket	114
$\binom{n}{k}$	binomial coefficient $n$ choose $k$	39
$K * L$	join of simplicial complexes	152

### 1.6 Vector spaces and vector fields

Symbol	Description	Page
$V/U$	quotient space	6
$V^*$	dual space	8
$L^2(\mathbb{R})$	square integrable functions on $\mathbb{R}$	27
$T_p M$	tangent space to $M$ at $p$	75
$X_p$	tangent vector at a point $p$	75
$X_p$	vector field $X$ evaluated at $p$	77
$T_p^* M$	cotangent space to $M$ at $p$	79
$X_f$	Hamiltonian vector field	114
$\Delta$	distribution	284

### 1.7 Matrices and maps

Symbol	Description	Page
$A^T$	transpose matrix	10
$A^{-1}$	inverse matrix	8
$W^\perp$	orthogonal complement of $W$	17
$\det \mathbf{T}$	determinant of matrix $\mathbf{T}$	17
$\det T$	determinant of map $T$	43
$A^*$	dual map	20
$A^\dagger$	adjoint map	20

$\mathbf{A}^\dagger$	adjoint matrix	20
$\mathbf{A}(i j)$	matrix $\mathbf{A}$ with row $i$ and column $j$ removed	22
$\tilde{A}_{ij}$	cofactor of $A_{ij}$	22
$\text{adj } \mathbf{A}$	adjugate matrix	22
$\text{tr } \mathbf{A}$	trace	24

### 1.8 Tensor components and tensor spaces

Symbol	Description	Page
$\delta_{ij}$	Kronecker delta	8
$\eta_{ij}$	Minkowski metric	19
$F_{ij}$	field strength	192
$g_{ij}$	generic metric	195
$\Gamma^a_{bc}$	Christoffel symbols (not a tensor)	202
$R^a_{bcd}$	Riemann curvature tensor	204
$R_{ij}$	Ricci curvature tensor	206
$R$	Ricci curvature scalar	207
$G_{ij}$	Einstein tensor	207
$T_{(i_1 \dots i_p)}$	symmetric part of $T$	36
$T_{[i_1 \dots i_p]}$	antisymmetric part of $T$	36
$a^{(i_1, \dots, i_p)}$	ascending indices	40
$a^I$	multi-index	40
$T^{i_1 \dots i_r}_{j_1 \dots j_s; k}$	semicolon notation	225
$\varepsilon_{i_1 \dots i_n}$	epsilon or Levi-Civita symbol	239
$\epsilon_{i_1 \dots i_n}$	epsilon or Levi-Civita (pseudo-)tensor	240
$\delta^{i_1 \dots i_k}_{j_1 \dots j_k}$	generalized Kronecker symbol	240
$\omega^i_j$	connection 1-forms	183
$\Omega^i_j$	curvature 2-forms	184
$\Omega(X, Y)$	curvature matrix	186
$R(X, Y)$	curvature operator	186
$K(\Pi)$	sectional curvature	232
$ds^2$	line element	197
$\tau$	torsion form/tensor	199
$\mathcal{T}^r$	tensors of order $r$	31

$\mathcal{T}_s^r$	tensors of type $(r, s)$	33
$\tilde{\mathcal{T}}_s^r$	multilinear maps of type $(r, s)$	35
$\text{Sym}^p V$	symmetric tensors of order $p$ on $V$	37
$\text{Alt}^p V$	antisymmetric tensors of order $p$ on $V$	37
$\bigwedge^p V$	$p$ -vectors on $V$	38
$\bigwedge V$	exterior algebra of $V$	41

### 1.9 Manifolds and bundles

Symbol	Description	Page
$S^2$	two sphere	54
$D^2$	two disk = two ball	148
$S^n$	$n$ -sphere	133
$B^n$	$n$ -ball	133
$T^2$	two torus	54
$TM$	tangent bundle of $M$	176
$T^*M$	cotangent bundle of $M$	176
$\Gamma(E)$	sections of bundle $E$	178
$M_p^k$	functions on $M$ vanishing to order $(k - 1)$ at $p$	81
$(E, M, Y, \pi)$	vector bundle	176
$g_{UV}$	vector bundle transition functions	178

### 1.10 Lie groups and Lie algebras

Symbol	Description	Page
$GL(n, \mathbb{R})$	general linear group (over the real field)	105
$SL(n, \mathbb{R})$	special linear group (over the real field)	235
$GL(n, \mathbb{C})$	general linear group (over the complex field)	108
$PGL(n, \mathbb{C})$	projective general linear group (over the complex field)	235
$M_n(\mathbb{R})$	set of (real) $n \times n$ matrices	105
$M_n^+(\mathbb{R})$	set of (real) symmetric $n \times n$ matrices	105
$O(n)$	orthogonal group	23
Exp	exponential map (on Lie group)	107
Exp	exponential map (on Riemannian manifold)	276

$\mathfrak{gl}(n, \mathbb{R})$	Lie algebra of $GL(n, \mathbb{R})$	107
$\mathfrak{gl}(n, \mathbb{C})$	Lie algebra of $GL(n, \mathbb{C})$	108
$SO(n)$	special orthogonal group	106
$\mathfrak{so}(n)$	Lie algebra of $SO(n)$	108
$U(n)$	unitary group	108
$SU(n)$	special unitary group	108
$\mathfrak{su}(n)$	Lie algebra of $SU(n)$	108

### 1.11 Homotopy, homology, cohomology, and holonomy

Symbol	Description	Page
$\pi_1(X, p)$	fundamental group of $X$ based at $p$	136
$\Omega^k(M)$	space of $k$ -forms on $M$	87
$Z^k(M)$	space of closed $k$ -forms on $M$	122
$B^k(M)$	space of exact $k$ forms on $M$	122
$H_{\text{dR}}^k(M)$	$k^{\text{th}}$ de Rham cohomology group on $M$	123
$\bar{\sigma}, \langle p_0, p_1, \dots \rangle$	oriented simplex	142
$C_m(K)$	$m$ -chains of simplicial complex $K$	143
$Z_m(K)$	$m$ -cycles of simplicial complex $K$	145
$B_m(K)$	$m$ -boundaries of simplicial complex $K$	145
$H_m(K)$	$m^{\text{th}}$ simplicial homology group of $K$	145
$\beta_m$	$m^{\text{th}}$ Betti number	145
$S_m(X)$	singular $m$ -chains of space $X$	146
$Z_m(X)$	singular $m$ -cycles of space $X$	147
$B_m(X)$	singular $m$ -boundaries of space $X$	147
$H_m(X)$	singular $m^{\text{th}}$ homology group of space $X$	147
$\chi(X)$	Euler characteristic of space $X$	149
$\sigma, (\bar{\sigma}, U, \phi)$	smooth singular simplex	158
$C_m^\infty(M)$	smooth singular $m$ -chains on $M$	158
$Z_m^\infty(M)$	smooth singular $m$ -cycles on $M$	159
$B_m^\infty(M)$	smooth singular $m$ -boundaries on $M$	159
$H_m^\infty(M)$	$m^{\text{th}}$ smooth singular homology group of $M$	159
$C^m(M)$	smooth singular $m$ -cochains of $M$	170
$Z^m(M)$	smooth singular $m$ -cocycles of $M$	170



$B^m(M)$	smooth singular $m$ -coboundaries of $M$	171
$H^m(M)$	smooth singular $m^{\text{th}}$ cohomology group of $M$	171
$\vartheta_t$	parallel transport map	213
$\mathcal{P}$	path ordering operator	218
$H(\nabla; p)$	holonomy group of $\nabla$ based at $p$	219
$\tilde{H}(\nabla; p)$	restricted holonomy group of $\nabla$ based at $p$	219