PHYSICS 121 FALL 2003 - Homework #1 - Solutions

Problems from Chapter 1: 3E, 7P, 21P
3. Using the given conversion factors, we find

(a) the distance \(d\) in rods to be

\[
d = 4.0 \text{ furlongs} \cdot \frac{201.168 \text{ m/furlong}}{5.0292 \text{ m/rod}} = 160 \text{ rods},
\]

(b) and that distance in chains to be

\[
d = \frac{(4.0 \text{ furlongs})(201.168 \text{ m/furlong})}{20.117 \text{ m/chain}} = 40 \text{ chains}.
\]
7. The volume of ice is given by the product of the semicircular surface area and the thickness. The semicircle area is \( A = \frac{\pi r^2}{2} \), where \( r \) is the radius. Therefore, the volume is

\[
V = \frac{\pi}{2} r^2 z
\]

where \( z \) is the ice thickness. Since there are \( 10^3 \) m in 1 km and \( 10^2 \) cm in 1 m, we have

\[
r = (2000 \text{ km})(\frac{10^3 \text{ m}}{1 \text{ km}})(\frac{10^2 \text{ cm}}{1 \text{ m}}) = 2000 \times 10^5 \text{ cm}.
\]

In these units, the thickness becomes

\[
z = (3000 \text{ m})(\frac{10^2 \text{ cm}}{1 \text{ m}}) = 3000 \times 10^2 \text{ cm}.
\]

Therefore,

\[
V = \frac{\pi}{2} \left(2000 \times 10^5 \text{ cm}\right)^2 \left(3000 \times 10^2 \text{ cm}\right) = 1.9 \times 10^{22} \text{ cm}^3.
\]
21. We introduce the notion of density (which the students have probably seen in other courses):

\[ \rho = \frac{m}{V} \]

and convert to SI units: 1 g = \(1 \times 10^{-3}\) kg.

(a) For volume conversion, we find 1 cm\(^3\) = \((1 \times 10^{-2}\) m\(^3\) = \(1 \times 10^{-6}\) m\(^3\). Thus, the density in kg/m\(^3\) is

\[ 1\text{ g/cm}^3 = \left( \frac{1\text{ g}}{\text{cm}^3} \right) \left( \frac{10^{-3}\text{ kg}}{\text{g}} \right) \left( \frac{\text{cm}^3}{10^{-6}\text{ m}^3} \right) = 1 \times 10^3 \text{ kg/m}^3. \]

Thus, the mass of a cubic meter of water is 1000 kg.

(b) We divide the mass of the water by the time taken to drain it. The mass is found from \(M = \rho V\) (the product of the volume of water and its density):

\[ M = (5700\text{ m}^3)(1 \times 10^3\text{ kg/m}^3) = 5.70 \times 10^6 \text{ kg}. \]

The time is \(t = (10\text{ h})(3600\text{ s/h}) = 3.6 \times 10^4\) s, so the mass flow rate \(R\) is

\[ R = \frac{M}{t} = \frac{5.70 \times 10^6\text{ kg}}{3.6 \times 10^4\text{ s}} = 158 \text{ kg/s}. \]
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Problems from Chapter 2: 1E, 5P a)&b), 13P, 17E, 23E, 37P, 43E, 48P
1. Assuming the horizontal velocity of the ball is constant, the horizontal displacement is

$$\Delta x = v \Delta t$$

where $\Delta x$ is the horizontal distance traveled, $\Delta t$ is the time, and $v$ is the (horizontal) velocity. Converting $v$ to meters per second, we have $160 \text{km/h} = 44.4 \text{m/s}$. Thus

$$\Delta t = \frac{\Delta x}{v} = \frac{18.4 \text{m}}{44.4 \text{m/s}} = 0.414 \text{s}.$$ 

The velocity-unit conversion implemented above can be figured “from basics” ($1000 \text{ m} = 1 \text{ km}, 3600 \text{ s} = 1 \text{ h}$) or found in Appendix D.
5. (a) Denoting the travel time and distance from San Antonio to Houston as $T$ and $D$, respectively, the average speed is

$$s_{\text{avg}_1} = \frac{D}{T} = \frac{(55 \text{ km/h}) \frac{T}{2} + (90 \text{ km/h}) \frac{T}{2}}{T} = 72.5 \text{ km/h}$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$s_{\text{avg}_2} = \frac{D}{T} = \frac{D}{\frac{D/2}{55 \text{ km/h}} + \frac{D/2}{90 \text{ km/h}}} = 68.3 \text{ km/h}$$

which should be rounded to 68 km/h.
13. Since \( v = \frac{dx}{dt} \) (Eq. 2-4), then \( \Delta x = \int v \, dt \), which corresponds to the area under the \( v \) vs \( t \) graph. Dividing the total area \( A \) into rectangular (base \( \times \) height) and triangular (\( \frac{1}{2} \) base \( \times \) height) areas, we have

\[
A = A_{0 < t < 2} + A_{2 < t < 10} + A_{10 < t < 12} + A_{12 < t < 16} \\
= \frac{1}{2}(2)(8) + (8)(8) + \left( (2)(4) + \frac{1}{2}(2)(4) \right) + (4)(4)
\]

with SI units understood. In this way, we obtain \( \Delta x = 100 \) m.
17. We represent its initial direction of motion as the +x direction, so that \( v_0 = +18 \text{ m/s} \) and \( v = -30 \text{ m/s} \) (when \( t = 2.4 \text{ s} \)). Using Eq. 2-7 (or Eq. 2-11, suitably interpreted) we find

\[
a_{avg} = \frac{(-30) - (+18)}{2.4} = -20 \text{ m/s}^2
\]

which indicates that the average acceleration has magnitude 20 m/s\(^2\) and is in the opposite direction to the particle’s initial velocity.
23. The constant-acceleration condition permits the use of Table 2-1.

(a) Setting $v = 0$ and $x_0 = 0$ in $v^2 = v_0^2 + 2a(x - x_0)$, we find

$$x = -\frac{1}{2} \frac{v_0^2}{a} = -\frac{1}{2} \left( \frac{5.00 \times 10^6}{-1.25 \times 10^{14}} \right) = 0.100 \text{ m}.$$ 

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time-plots of the position $x$ and velocity $v$ of the muon from the moment it enters the field to the time it stops. The computation in part (a) made no reference to $t$, so that other equations from Table 2-1 (such as $v = v_0 + at$ and $x = v_0 t + \frac{1}{2} at^2$) are used in making these plots.
37. We denote $t_r$ as the reaction time and $t_b$ as the braking time. The motion during $t_r$ is of the constant-velocity (call it $v_0$) type. Then the position of the car is given by

$$x = v_0 t_r + v_0 t_b + \frac{1}{2}at^2_b$$

where $v_0$ is the initial velocity and $a$ is the acceleration (which we expect to be negative-valued since we are taking the velocity in the positive direction and we know the car is decelerating). After the brakes are applied the velocity of the car is given by $v = v_0 + at_b$. Using this equation, with $v = 0$, we eliminate $t_b$ from the first equation and obtain

$$x = v_0 t_r - \frac{v_0^2}{a} + \frac{1}{2}\frac{v_0^2}{a} = v_0 t_r - \frac{1}{2}\frac{v_0^2}{a}.$$

We write this equation for each of the initial velocities:

$$x_1 = v_0 t_r - \frac{1}{2}\frac{v_{01}^2}{a}$$

and

$$x_2 = v_0 t_r - \frac{1}{2}\frac{v_{02}^2}{a}.$$

Solving these equations simultaneously for $t_r$ and $a$ we get

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02}(v_{02} - v_{01})}$$

and

$$a = -\frac{1}{2} \frac{v_{02} v_{01}^2 - v_{01} v_{02}^2}{v_{02} x_1 - v_{01} x_2}.$$

Substituting $x_1 = 56.7\text{ m}$, $v_{01} = 80.5\text{ km/h} = 22.4\text{ m/s}$, $x_2 = 24.4\text{ m}$ and $v_{02} = 48.3\text{ km/h} = 13.4\text{ m/s}$, we find

$$t_r = \frac{13.4^2(56.7) - 22.4^2(24.4)}{(22.4)(13.4)(13.4 - 22.4)} = 0.74\text{ s}$$

and

$$a = -\frac{1}{2} \frac{(13.4)22.4^2 - (22.4)13.4^2}{(13.4)(56.7) - (22.4)(24.4)} = -6.2\text{ m/s}^2.$$

The magnitude of the deceleration is therefore $6.2\text{ m/s}^2$. Although rounded off values are displayed in the above substitutions, what we have input into our calculators are the “exact” values (such as $v_{02} = \frac{161}{12}\text{ m/s}$).
43. We neglect air resistance for the duration of the motion (between “launching” and “landing”), so \( a = -g = -9.8 \text{ m/s}^2 \) (we take downward to be the \(-y\) direction). We use the equations in Table 2-1 (with \( \Delta y \) replacing \( \Delta x \)) because this is \( a = \text{constant motion} \).

(a) At the highest point the velocity of the ball vanishes. Taking \( y_0 = 0 \), we set \( v = 0 \) in \( v^2 = v_0^2 - 2gy \), and solve for the initial velocity: \( v_0 = \sqrt{2gy} \). Since \( y = 50 \text{ m} \) we find \( v_0 = 31 \text{ m/s} \).

(b) It will be in the air from the time it leaves the ground until the time it returns to the ground \((y = 0)\). Applying Eq. 2-15 to the entire motion (the rise and the fall, of total time \( t > 0 \)) we have

\[
y = v_0t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{2v_0}{g}
\]

which (using our result from part (a)) produces \( t = 6.4 \text{ s} \). It is possible to obtain this without using part (a)’s result; one can find the time just for the rise (from ground to highest point) from Eq. 2-16 and then double it.

(c) SI units are understood in the \( x \) and \( v \) graphs shown. In the interest of saving space, we do not show the graph of \( a \), which is a horizontal line at \(-9.8 \text{ m/s}^2\).
48. We neglect air resistance, which justifies setting \( a = -g = -9.8 \text{ m/s}^2 \) (taking down as the \(-y\) direction) for the duration of the motion. We are allowed to use Table 2-1 (with \( \Delta y \) replacing \( \Delta x \)) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the \( y \) axis. The total time of fall can be computed from Eq. 2-15 (using the quadratic formula).

\[
\Delta y = v_0 t - \frac{1}{2}gt^2 \implies t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}
\]

with the positive root chosen. With \( y = 0, v_0 = 0 \) and \( y_0 = h = 60 \text{ m} \), we obtain

\[
t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}} = 3.5 \text{ s}.
\]

Thus, “1.2 s earlier” means we are examining where the rock is at \( t = 2.3 \text{ s} \):

\[
y - h = v_0(2.3) - \frac{1}{2}g(2.3)^2 \implies y = 34 \text{ m}
\]

where we again use the fact that \( h = 60 \text{ m} \) and \( v_0 = 0 \).
Additional Problems:

I. If a runner runs at an average speed of 7.0 miles/hour while the runner’s heart beats at 100 beats/minute, how many times does the runner’s heart beat during a 3.0 mile run?

The time it takes the runner to run 3.0 miles is
\[ \Delta t = \frac{D}{s_{\text{avg}}} = \frac{3.0 \text{ mile}}{7.0 \text{ mile/hour}} = 0.4286 \text{ hour} \times (60 \text{ min/hour}) = 25.71 \text{ min} \]

The number of heartbeats in 25.71 min is
\[ \# \text{of beats} = (100 \text{ beats/min}) \times 25.71 \text{ min} = 2571 \text{ beats} \]
\[ \approx 2600 \text{ beats} \quad \text{ rounded to 2 sig. figs.} \]

II. A mouse moving at a constant velocity of 3.0 m/s passes a cat at rest. The cat immediately begins chasing the mouse moving with constant acceleration. If the cat catches the mouse a distance of 4.0 m from its starting point,

a. How long (how much time) does it take for the cat to catch the mouse?

The time it takes for the mouse (and the cat) to move 4.0 m is
\[ \Delta t = \frac{\Delta x}{v_{\text{avg}}} = \frac{4.0 \text{ m}}{3.0 \text{ m/s}} = 1.333 \text{ s} \]
\[ \approx 1.3 \text{ s} \quad \text{ rounded to 2 sig figs} \]

b. What is the cat's acceleration?

The cat is moving with constant acceleration so the cat's position is given by:
\[ x = x_o + v_o t + \frac{1}{2} a t^2 \]
you can assume that \( x_o = 0 \). The cat is starting from rest so, \( v_o = 0 \) so:
\[ x = \frac{1}{2} a t^2 \]
Solving for \( a \):
\[ a = 2 x / t^2 \]
We already know that when \( x = 4.0 \text{ m} \) \( t = 1.333 \text{ s} \) so:
\[ a = \frac{2(4.0 \text{ m})}{(1.333 \text{ s})^2} = 4.502 \text{ m/s}^2 \]
\[ \approx 4.5 \text{ m/s}^2 \quad \text{ rounded to 2 sig figs} \]

c. What is the average speed of the cat while it is chasing the mouse?

The cat and mouse undergo the same displacement in the same time. So the average velocity of the cat must be the same as the velocity of the mouse:
\[ s_{\text{avg}} = 3.0 \text{ m/s} \]
III. In problem 7P in Chapter 1, you found that the volume of ice in Antarctica is $1.9 \times 10^{22} \text{ cm}^3$. If all of the ice in Antarctica were to melt, how many meters would sea level rise. You will need the following information: The density of ice is 920 kg/m$^3$ and the density of liquid water is 1000 kg/m$^3$. The total surface area of the Earth's oceans is $3.60 \times 10^8 \text{ km}^2$.

The mass of the ice is: $M = \text{ice density} \times \text{ice volume}$

The mass of the melted water is the same as that of the ice:

$M = (\text{water density})(\text{water volume})$

The volume of the water is given by:

water volume = (area of oceans)(rise in sea level)

Solving for rise in sea level:

$\text{rise in sea level} = \frac{\text{water volume}}{\text{(area of oceans)}}$

$= \frac{(\text{M}/(\text{water density}))}{(\text{area of oceans})}$

$= \frac{(\text{ice density})/(\text{water density})(\text{ice volume})}{(\text{area of oceans})}$

$= \frac{920 \text{ kg/m}^3/1000 \text{ kg/m}^3}{(1.9 \times 10^{22} \text{ cm}^3)/(3.60 \times 10^8 \text{ km}^2)}$

$= 48.56 \text{ m}$

$= 49 \text{ m}$ rounded to 2 sig figs