PHYSICS 121 FALL 2003 - Homework #2 - Solutions

Problems from Chapter 3: 3E, 5E, 15E, 31P
3. The $x$ component of $\vec{a}$ is given by $a_x = 7.3 \cos 250^\circ = -2.5$ and the $y$ component is given by $a_y = 7.3 \sin 250^\circ = -6.9$. In considering the variety of ways to compute these, we note that the vector is $70^\circ$ below the $-x$ axis, so the components could also have been found from $a_x = -7.3 \cos 70^\circ$ and $a_y = -7.3 \sin 70^\circ$. In a similar vein, we note that the vector is $20^\circ$ from the $-y$ axis, so one could use $a_x = -7.3 \sin 20^\circ$ and $a_y = -7.3 \cos 20^\circ$ to achieve the same results.
5. The textbook’s approach to this sort of problem is through the use of Eq. 3-6, and is illustrated in Sample Problem 3-3. However, most modern graphical calculators can produce the results quite efficiently using rectangular ↔ polar “shortcuts.”

(a) \( \sqrt{(-25)^2 + 40^2} = 47.2 \text{ m} \)

(b) Recalling that \( \tan(\theta) = \tan(\theta + 180°) \), we note that the two possibilities for \( \tan^{-1} \left( \frac{40}{-25} \right) \) are \(-58°\) and \(122°\). Noting that the vector is in the third quadrant (by the signs of its \( x \) and \( y \) components) we see that \(122°\) is the correct answer. The graphical calculator “shortcuts” mentioned above are designed to correctly choose the right possibility.
15. The vectors are shown on the diagram. The $x$ axis runs from west to east and the $y$ axis runs from south to north. Then $a_x = 5.0 \text{ m}$, $a_y = 0$, $b_x = -(4.0 \text{ m}) \sin 35^\circ = -2.29 \text{ m}$, and $b_y = (4.0 \text{ m}) \cos 35^\circ = 3.28 \text{ m}$.

(a) Let $\vec{c} = \vec{a} + \vec{b}$. Then $c_x = a_x + b_x = 5.0 \text{ m} - 2.29 \text{ m} = 2.71 \text{ m}$ and $c_y = a_y + b_y = 0 + 3.28 \text{ m} = 3.28 \text{ m}$. The magnitude of $\vec{c}$ is

$$c = \sqrt{c_x^2 + c_y^2} = \sqrt{(2.71 \text{ m})^2 + (3.28 \text{ m})^2} = 4.3 \text{ m}.$$  

(b) The angle $\theta$ that $\vec{c} = \vec{a} + \vec{b}$ makes with the $+x$ axis is

$$\theta = \tan^{-1} \frac{c_y}{c_x} = \tan^{-1} \frac{3.28 \text{ m}}{2.71 \text{ m}} = 50.4^\circ.$$  

The second possibility ($\theta = 50.4^\circ + 180^\circ = 126^\circ$) is rejected because it would point in a direction opposite to $\vec{c}$.

(c) The vector $\vec{b} - \vec{a}$ is found by adding $-\vec{a}$ to $\vec{b}$. The result is shown on the diagram to the right. Let $\vec{c} = \vec{b} - \vec{a}$. Then $c_x = b_x - a_x = -2.29 \text{ m} - 5.0 \text{ m} = -7.29 \text{ m}$ and $c_y = b_y - a_y = 3.28 \text{ m}$. The magnitude of $\vec{c}$ is $c = \sqrt{c_x^2 + c_y^2} = 8.0 \text{ m}$.

(d) The tangent of the angle $\theta$ that $\vec{c}$ makes with the $+x$ axis (east) is

$$\tan \theta = \frac{c_y}{c_x} = \frac{3.28 \text{ m}}{-7.29 \text{ m}} = -4.50.$$  

There are two solutions: $-24.2^\circ$ and $155.8^\circ$. As the diagram shows, the second solution is correct. The vector $\vec{c} = -\vec{a} + \vec{b}$ is $24^\circ$ north of west.
31. Since \( ab \cos \phi = a_x b_x + a_y b_y + a_z b_z \),

\[
\cos \phi = \frac{a_x b_x + a_y b_y + a_z b_z}{ab}.
\]

The magnitudes of the vectors given in the problem are

\[
a = |\vec{a}| = \sqrt{(3.0)^2 + (3.0)^2 + (3.0)^2} = 5.2
\]

\[
b = |\vec{b}| = \sqrt{(2.0)^2 + (1.0)^2 + (3.0)^2} = 3.7
\]

The angle between them is found from

\[
\cos \phi = \frac{(3.0)(2.0) + (3.0)(1.0) + (3.0)(3.0)}{(5.2)(3.7)} = 0.926.
\]

The angle is \( \phi = 22^\circ \).
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Problems from Chapter 4: 4P, 8P, 11E a), b) & c), 17E, 18E, 47P
4. We use a coordinate system with $+x$ eastward and $+y$ upward. We note that $123^\circ$ is the angle between the initial position and later position vectors, so that the angle from $+x$ to the later position vector is $40^\circ + 123^\circ = 163^\circ$. In unit-vector notation, the position vectors are

$$\vec{r}_1 = 360 \cos(40^\circ) \hat{i} + 360 \sin(40^\circ) \hat{j} = 276 \hat{i} + 231 \hat{j}$$
$$\vec{r}_2 = 790 \cos(163^\circ) \hat{i} + 790 \sin(163^\circ) \hat{j} = -755 \hat{i} + 231 \hat{j}$$

respectively (in meters). Consequently, we plug into Eq. 4-3

$$\Delta \vec{r} = ((-755) - 276) \hat{i} + (231 - 231) \hat{j}$$

and find the displacement vector is horizontal (westward) with a length of 1.03 km. If unit-vector notation is not a priority in this problem, then the computation can be approached in a variety of ways – particularly in view of the fact that a number of vector capable calculators are on the market which reduce this problem to a very few keystrokes (using magnitude-angle notation throughout).
8. On the one hand, we could perform the vector addition of the displacements with a vector capable calculator in polar mode \((\theta_1 \triangleq 37^\circ) + (\theta_2 \triangleq -90^\circ) = (\theta_3 \triangleq -18^\circ)\), but in keeping with Eq. 3-5 and Eq. 3-6 we will show the details in unit-vector notation. We use a 'standard' coordinate system with +x East and +y North. Lengths are in kilometers and times are in hours.

(a) We perform the vector addition of individual displacements to find the net displacement of the camel.

\[
\begin{align*}
\Delta r_1 &= 75 \cos(37^\circ) \hat{i} + 75 \sin(37^\circ) \hat{j} \\
\Delta r_2 &= -65 \hat{j} \\
\Delta r_1 + \Delta r_2 &= 60 \hat{i} - 20 \hat{j} \text{ km}.
\end{align*}
\]

If it is desired to express this in magnitude-angle notation, then this is equivalent to a vector of length \(\sqrt{60^2 + (-20)^2} = 63 \text{ km}\), which is directed at 18° south of east.

(b) We use the result from part (a) in Eq. 4-8 along with the fact that \(\Delta t = 90 \text{ h}\). In unit vector notation, we obtain

\[
\vec{v}_{\text{avg}} = \frac{60 \hat{i} - 20 \hat{j}}{90} = 0.66 \hat{i} - 0.22 \hat{j}
\]

in kilometers-per-hour. This result in magnitude-angle notation is \(\vec{v}_{\text{avg}} = 0.70 \text{ km/h} \text{ at } 18^\circ \text{ south of east}\).

(c) Average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. Since the camel travels 140 km, we obtain \(140/90 = 1.56 \text{ km/h}\).

(d) The net displacement is required to be the 90 km East from \(A\) to \(B\). The displacement from the resting place to \(B\) is denoted \(\vec{r}_3\). Thus, we must have (in kilometers)

\[
\vec{r}_1 + \vec{r}_2 + \vec{r}_3 = 90 \hat{i}
\]

which produces \(\vec{r}_3 = 30 \hat{i} + 20 \hat{j}\) in unit-vector notation, or \((36 \triangleq 33^\circ)\) in magnitude-angle notation. Therefore, using Eq. 4-8 we obtain

\[
|\vec{v}_{\text{avg}}| = \frac{36 \text{ km}}{120 - 90 \text{ h}} = 1.2 \text{ km/h}
\]

and the direction of this vector is the same as \(\vec{r}_3\) (that is, 33° north of east).
11. In parts (b) and (c), we use Eq. 4-10 and Eq. 4-16. For part (d), we find the direction of the velocity computed in part (b), since that represents the asked-for tangent line.

(a) Plugging into the given expression, we obtain
\[ \vec{r} \bigg|_{t=2} = (2(8) - 5(2)) \hat{i} + (6 - 7(16)) \hat{j} = 6.00 \hat{i} - 106 \hat{j} \]
in meters.

(b) Taking the derivative of the given expression produces
\[ \vec{v}(t) = (6.00t^2 - 5.00) \hat{i} + 28.0t^3 \hat{j} \]
where we have written \( v(t) \) to emphasize its dependence on time. This becomes, at \( t = 2.00 \) s, \( \vec{v} = 19.0 \hat{i} - 224 \hat{j} \text{ m/s} \).

(c) Differentiating the \( \vec{v}(t) \) found above, with respect to \( t \) produces \( 12.0t \hat{i} - 84.0t^2 \hat{j} \), which yields \( \vec{a} = 24.0 \hat{i} - 336 \hat{j} \text{ m/s}^2 \) at \( t = 2.00 \) s.

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18. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the initial point (edge of table), the \( y \) coordinate of the ball is given by \( y = -\frac{1}{2}gt^2 \). If \( t \) is the time of flight and \( y = -1.20 \text{ m} \) indicates the level at which the ball hits the floor, then

\[
t = \sqrt{\frac{2(1.20)}{9.8}} = 0.495 \text{ s}.
\]

(b) The initial (horizontal) velocity of the ball is \( \vec{v} = v_0 \hat{i} \). Since \( x = 1.52 \text{ m} \) is the horizontal position of its impact point with the floor, we have \( x = v_0 t \). Thus,

\[
v_0 = \frac{x}{t} = \frac{1.52}{0.495} = 3.07 \text{ m/s}.
\]
47. The radius of Earth may be found in Appendix C.

(a) The speed of a person at Earth’s equator is \( v = \frac{2\pi R}{T} \), where \( R \) is the radius of Earth (6.37 × 10^6 m) and \( T \) is the length of a day (8.64 × 10^4 s): \( v = \frac{2\pi (6.37 \times 10^6 \text{ m})}{(8.64 \times 10^4 \text{ s})} = 463 \text{ m/s} \). The magnitude of the acceleration is given by

\[
a = \frac{v^2}{R} = \frac{(463 \text{ m/s})^2}{6.37 \times 10^6 \text{ m}} = 0.034 \text{ m/s}^2.
\]

(b) If \( T \) is the period, then \( v = \frac{2\pi R}{T} \) is the speed and \( a = \frac{v^2}{R} = \frac{4\pi^2 R^2}{T^2 R} = \frac{4\pi^2 R}{T^2} \) is the magnitude of the acceleration. Thus

\[
T = 2\pi \sqrt{\frac{R}{a}} = 2\pi \sqrt{\frac{6.37 \times 10^6 \text{ m}}{9.8 \text{ m/s}^2}} = 5.1 \times 10^3 \text{ s} = 84 \text{ min}.
\]
Additional Problems:

I. A cat starts from the position, \( x = 10.0 \text{ m} \) (East of the origin), \( y = 5.0 \text{ m} \) (North of the origin). The cat then undergoes a displacement with a magnitude of 3.0 m in the direction 30° West of North.

   a. What are the \( x \) and \( y \) coordinates of the cat’s final position?

   \[
   x_f = x_i + \Delta x
   \]
   \[
   y_f = y_i + \Delta y
   \]
   \[
   \Delta x = -3.0 \text{ m} \sin(30°) = -1.5 \text{ m}
   \]
   \[
   \Delta y = 3.0 \text{ m} \cos(30°) = 2.5981 \text{ m}
   \]
   \[
   x_f = 10.0 \text{ m} - 1.5 \text{ m} = 8.5 \text{ m}
   \]
   \[
   y_f = 5.0 \text{ m} + 2.5981 \text{ m} = 7.5981 \text{ m}
   \]
   \[
   x_f = 8.5 \text{ m}
   \]
   \[
   y_f = 7.6 \text{ m}
   \]

   b. What are the polar coordinates, \( r \) and \( \theta \), of the cat’s final position?

   \[
   r = \sqrt{x_f^2 + y_f^2} = \sqrt{(8.5)^2 + (7.5981)^2} = 11.40 \text{ m}
   \]
   \[
   r = 11 \text{ m}
   \]
   \[
   \theta = \tan^{-1}(7.5981/8.5) = 41.79°
   \]
   \[
   \theta = 42° \text{ North of East} = 48° \text{ East of North}
   \]

   c. If the cat’s displacement takes 2.0 s, what is the magnitude and direction of the cat’s average velocity?

   magnitude:
   \[
   |\vec{v}_{avg}| = \frac{|\Delta \vec{r}|}{\Delta t} = \frac{3.0 \text{ m}}{2.0 \text{ s}} = 1.5 \text{ m/s}
   \]
   \[
   |\vec{v}_{avg}| = 1.5 \text{ m/s}
   \]

   direction: The direction of the average velocity is the same as the displacement
   \[
   30° \text{ West of North} \text{ or } 60° \text{ North of West}
   \]
II. A car attempts to jump a chasm. The chasm is 30.0 m wide, and the side that the car jumps from is 20.0 m higher than the other side. How fast must the car be going when drives over the edge of the chasm in order to make the jump? Assume that the car’s velocity is in the horizontal direction when it leaves the higher side.

\[ \Delta x = 30.0 \text{ m}, \quad \Delta y = -20.0 \text{ m} \]

\[ \Delta t = \frac{\Delta x}{v_x} \quad \text{and} \quad \Delta y = -\frac{1}{2}g(\Delta t)^2 \]

so \[ \Delta y = -\frac{1}{2}g(\Delta x/v_x)^2 \]

which yields,

\[ v_x = \sqrt{\frac{-g(\Delta x)^2}{2\Delta y}} = \sqrt{\frac{-(9.8 \text{ m/s}^2)(30.0 \text{ m})^2}{2(-20.0 \text{ m})}} = 14.849 \text{ m/s} \]

Since the car is moving horizontally when it leaves the edge, \( v_i = v_x \)

so \( v_i = 14.8 \text{ m/s} \)

III. The Earth orbits the Sun once per year. The distance from the Earth to the Sun is \( 1.5 \times 10^{11} \) m. If the Earth’s orbit is uniform circular motion,

a. What is the speed of the Earth as it revolves around the Sun?

\[ v = \frac{2\pi r}{T} = \frac{2\pi(1.5 \times 10^{11} \text{ m})}{(1.00 \text{ year})} \]

\[ = \frac{2\pi(1.5 \times 10^{11} \text{ m})}{(1.00 (365.25 \times 24 \times 60 \times 60 \text{ s}))} = 29865 \text{ m/s} \]

\[ v = 30,000 \text{ m/s} = 3.0 \times 10^4 \text{ m/s} \]

b. What is the centripetal acceleration of the Earth as it revolves around the Sun?

\[ a_c = \frac{v^2}{r} = \frac{(29865 \text{ m/s})^2}{(1.5 \times 10^{11} \text{ m})} = 0.0059462 \text{ m/s}^2 \]

\[ a_c = 0.0059 \text{ m/s}^2 = 5.9 \times 10^{-3} \text{ m/s}^2 \]